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A NOTE ON AXISYMMETRICAL FLUTTER OF CIRCULAR CYLINDRICAL SHELLS OF FINITE LENGTH

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IN [1], Krumhaar investigated the stability of axisymmetrical oscillations of a circular cylindrical shell of finite length in a supersonic airstream parallel to the cylinder axis; he studied the corresponding eigenvalue problem, calculated the eigenvalues and derived the exact flutter boundaries. (A detailed survey on flutter problems is given in [2].) In the present note, the mode shapes of axisymmetrical oscillations in a neighbourhood of these flutter boundaries are investigated.

By restricting to radial oscillations of the form $\hat{w}(\xi, t) = w(\xi)\,e^{i\omega t}$, the stability problem leads to the following non-self-adjoint eigenvalue problem:

$$\frac{d^4w}{d\xi^4}+A\frac{dw}{d\xi} = \lambda w(\xi), \qquad 0 \le \xi \le 1 \tag{1a}$$

$$w(0) = w(1) = \left(\frac{d^2w}{d\xi^2}\right)_{\xi=0} = \left(\frac{d^2w}{d\xi^2}\right)_{\xi=1} = 0 \tag{1b}$$

Here $\xi = x/L$ is a nondimensional variable, x being the coordinate along the generators of the shell and L its length (see Fig. 1); $w(\xi)$ denotes the complex amplitude of the shell

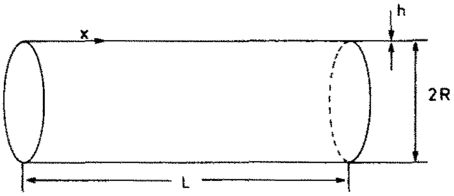

FIG. 1. Sketch of the cylindrical shell.

vibration in radial direction, A is a generalized velocity proportional to the Mach number M, and λ is a complex eigenvalue parameter. As far as the distribution, multiplicity and dependence on the real parameter A are concerned, the eigenvalue problem (1) has been investigated in [1] for $0 \le A \le 1{\cdot}1\times10^5$. The axisymmetrical shell oscillations are stable as long as the generalized velocity A remains below a certain critical value A_c, so that all corresponding eigenvalues $\lambda_j(A)$, $j = 1, 2, \ldots$ are located inside the so-called stability parabola; the motion is unstable, however, if $A > A_c$, so that at least one of the corresponding eigenvalues $\lambda_j(A)$, $j = 1, 2, \ldots$ is located outside the stability parabola. It was further pointed out in [1] that at least in the range mentioned above the eigenvalue $\lambda_1(A)$ (i.e. the eigenvalue coinciding with π^4 for $A = 0$) will be the first one to cross the stability parabola for increasing A. The critical value A_c is, therefore, defined to be that value of A for which $\lambda_1(A)$ coincides with the stability parabola. M_c is the corresponding Mach number. The shape of this parabola is determined by the parameters specifying the problem (see [1], p. 30). An increase of the material damping expressed by the damping coefficient γ broadens the parabola if all other parameters are kept constant, and hence has a stabilizing influence on the flutter of the shell.

In this note we shall investigate the mode shape of the shell oscillation, i.e. the eigenfunction $w_1(\xi, A)$ belonging to the eigenvalue $\lambda_1(A)$ for some fixed value of A. For the case of a flat plate, which is described by the same eigenvalue problem but by a stability parabola of smaller width, and which, therefore, leads to considerably smaller values of the critical velocity A_c, Movchan [3] found that the maxima of the mode shapes are rapidly shifted towards the trailing edge with increasing velocity A. To compare the behaviour of a circular cylindrical shell with that of the flat plate, we are especially interested in the behaviour of the mode shapes $w_1(\xi, A)$ when A passes from stable values through A_c to unstable values.

For the calculation of $w_1(\xi, A)$ the eigenvalues given in [1; Fig. 7] are used. (Actually, the eigenvalues are not taken from [1; Fig. 7] but from the original computer data underlying this figure.) As (1a) is a differential equation with constant coefficients, $w_1(\xi, A)$ may be represented in the form

$$w_1(\xi, A) = \sum_{j=1}^{4} c_j \, \mathrm{e}^{-z_j \xi}$$

if the four roots z_j of

$$z^4 - Az - \lambda_1(A) = 0$$

are different. This condition is fulfilled for all numerical calculations referred to below.

The boundary conditions (1b) lead to a homogeneous system of four linear equations for c_j. Since its determinant is equal to zero, c_j may be calculated up to a common arbitrary factor:

$$\begin{aligned}
c_1 &= \mathrm{e}^{-z_2}(z_4^2 - z_3^2) + \mathrm{e}^{-z_3}(z_2^2 - z_1^2) + \mathrm{e}^{-z_4}(z_1^2 - z_2^2)\\
c_2 &= \mathrm{e}^{-z_1}(z_3^2 - z_4^2) + \mathrm{e}^{-z_3}(z_1^2 - z_4^2) + \mathrm{e}^{-z_4}(z_2^2 - z_1^2)\\
c_3 &= \mathrm{e}^{-z_1}(z_4^2 - z_2^2) + \mathrm{e}^{-z_2}(z_4^2 - z_1^2) + \mathrm{e}^{-z_4}(z_1^2 - z_3^2)\\
c_4 &= \mathrm{e}^{-z_1}(z_2^2 - z_3^2) + \mathrm{e}^{-z_2}(z_2^2 - z_4^2) + \mathrm{e}^{-z_3}(z_3^2 - z_2^2).
\end{aligned}$$

The numerical computation of $w_1(\xi, A)$ is related to an experimental situation described by the data given in [1; p. 50].

Calculations were carried through for several values of the damping coefficient γ and the ratio h/R of the shell thickness to the shell radius.

For the special combination $\gamma = 0{\cdot}0005$; $h/R = 6{\cdot}25 \times 10^{-4}$ one obtains $A_c = 97903{\cdot}37$ and $M_c = 1{\cdot}54$. For these values Fig. 2 represents in a suitable normalisation the real parts $\mathscr{R}w_1(\xi, A)$, plotted versus the nondimensional variable ξ in the interval $0{\cdot}5 \leq \xi \leq 1$, for two values of the parameter A belonging to the stable domain ($A < A_c$), for the critical value ($A = A_c$), and for two values of A belonging to the unstable domain ($A > A_c$). The interval $0 \leq \xi \leq 0{\cdot}5$ was omitted since $\mathscr{R}w_1(\xi, A)$ is very small there compared with its values in $0{\cdot}5 \leq \xi \leq 1$. One observes that the maxima of $\mathscr{R}w_1(\xi, A)$ are shifted towards the trailing edge of the shell with increasing values of A, i.e. with increasing velocity of the airstream. The imaginary part $\mathscr{I}w_1(\xi, A)$ displays a similar behaviour.

This result corresponds to what Movchan found in [3] for the case of a flat panel of finite length.

The computations using other values for γ and h/R yield qualitatively the same result as that mentioned above.

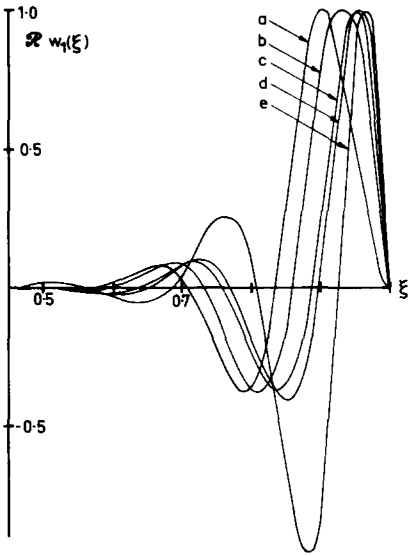

FIG. 2. Real parts $\mathscr{R}w_1(\xi, A)$ of the mode shapes for different Mach numbers and $\gamma = 0{\cdot}0005$, $h/R = 6{\cdot}25 \times 10^{-4}$. The graphs a, b, c, d, e correspond to the Mach numbers $M = 0{\cdot}94\,M_c$, $M = 0{\cdot}97\,M_c$, $M = M_c$, $M = 1{\cdot}01\,M_c$, $M = 1{\cdot}05\,M_c$ respectively.

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